

(考試時間 2 小時)

1. (25%)

The Great Force Motors Corporation (GFMC) is planning the introduction of a brand new SUV – the Vector. There are two options for production. One is to build the Vector at the company's existing plant in Indiana, sharing production time with its line of minivans that are currently being produced there. If sales of the Vector are just moderate, this will work out well as there is sufficient capacity to produce both types of vehicles at the same plant. However, if sales of the Vector are strong, this option would require the operation of a third shift, which would lead to significantly higher costs.

A second option is to open a new plant in Georgia. This plant would have sufficient capacity to meet even the largest projections for sales of the Vector. However, if sales are only moderate, the plant would be underutilized and therefore less efficient.

This is a new design, so sales are hard to predict. However, GFMC predicts that there would be about a 60% chance of strong sales (annual sales of 100,000), and a 40% chance of moderate sales (annual sales of 50,000). The average revenue per Vector sold is \$30,000. Production costs per vehicle for the two production options depend upon sales, as indicated in the table below.

	Moderate Sales	Strong Sales
Shared Plant in Indiana	16,000	24,000
Dedicated Plant in Georgia	22,000	20,000

The amortized annual cost of plant construction and other associated fixed costs for the Georgia plant would total \$400 million per year (regardless of sales volume). The fixed costs for adding Vector production to the plant in Indiana would total \$200 million per year (regardless of sales volume).

- Construct a decision tree to determine which production option maximizes the expected annual profit, considering fixed costs, production costs, and sales revenues.
- Due to the uncertainty in expected sales for the Vector, GFMC is considering conducting a marketing survey to determine customer attitudes toward the Vector and better predict the likelihood of strong sales. The marketing survey would give one of two results—a positive attitude or a negative attitude toward the design. GFMC has used this marketing survey for other vehicles. For vehicles that eventually had strong sales, the marketing survey indicated positive attitudes toward

the design 70% of the time and negative attitudes 30% of the time. For vehicles that eventually had moderate sales, the marketing survey indicated positive attitudes toward the design 20% of the time and negative attitudes 80% of the time. Assuming GFMC conducts such a survey, construct a decision tree to determine how the company should proceed and what the expected annual profit would be (ignoring the cost of the survey).

- c. What is the expected value of the sample information in part b? What does this say about how large the cost of the marketing survey can be before it would no longer be worthwhile to conduct the survey?

2. (25%)

Tony and Robert are two barbers in a barber shop they own and operate. They provide two chairs for customers who are waiting to begin a haircut, so the number of customers in the shop varies between 0 and 4. For $n = 0, 1, 2, 3, 4$, the probability P_n that exactly n customers are in the shop is $P_0 = 1/16, P_1 = 4/16, P_2 = 6/16, P_3 = 4/16, P_4 = 1/16$.

- a. Calculate L , the expected number of customers in the system. How would you describe the meaning of L to Tony and Robert?
- b. For each of the possible values of the number of customers in the queuing system, specify how many customers are in the queue. For each of the possible numbers in the queue, multiply by its probability, and then add the products to calculate L_q , the expected number of customers in the queue. How would you describe the meaning of L_q to Tony and Robert?
- c. Given that an average of four customers per hour arrive and stay to receive a haircut, determine W , the expected waiting time in the system, and W_q , the expected waiting time in the queue. Describe these two quantities in terms meaningful to Tony and Robert.
- d. Given that Tony and Robert are equally fast in giving haircuts, what is the average duration of a haircut?

3. (24 points) Consider the following problem.

$$\text{Maximize } Z = 2x_1 - 4x_2$$

$$\text{subject to } x_1 - x_2 \leq 1$$

$$\text{and } x_1 \geq 0, \quad x_2 \geq 0.$$

- Construct the dual problem, and then find its optimal solution by inspection.
- Use the complementary slackness property and the optimal solution for the dual problem to find the optimal solution for the primal problem.
- Suppose that c_j , the coefficient of x_j in the primal objective function, actually can have any value in the model. For what values of c_j does the dual problem have no feasible solutions? For these values, what does duality theory then imply about the primal problem?

4. (26 points) A U.S. professor will be spending a short sabbatical leave at the University of Iceland. She wishes to bring all needed items with her on the airplane. After collecting the professional items that she must have, she finds that airplane regulations on space and weight for checked luggage will severely limit the clothes she can take. (She plans to carry on a warm coat and then purchase a warm Icelandic sweater upon arriving in Iceland.) Clothes under consideration for checked luggage include 3 skirts, 3 slacks, 4 tops, and 3 dresses. The professor wants to maximize the number of outfits she will in Iceland (including the special dress she will have wear on the airplane). Each dress constitutes an outfit. Other outfits consist of a combination of a top and either a skirt or slacks. However, certain combinations are not fashionable and so will not qualify as an outfit.

In the following table, the combinations that will make an outfit are marked with an x.

		Top				Icelandic Sweater
		1	2	3	4	
Skirt	1	x	x			x
	2	x			x	
	3		x	x	x	x
Slacks	1	x		x		
	2	x	x		x	x
	3			x	x	x

The weight (in grams) and volume (in cubic centimeters) of each item are shown in the

following table:

		Weight	Volume
Skirt	1	600	5000
	2	450	3500
	3	700	3000
Slacks	1	600	3500
	2	550	6000
	3	500	4000
Top	1	350	4000
	2	300	3500
	3	300	3000
	4	450	5000
Dress	1	600	6000
	2	700	5000
	3	800	4000
Total allowed		4000	32000

Formulate a BIP model to choose which items of clothing to take?

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