（考試時間 2 小時）

1. (20%)

Consider a network which is composed of "M" nodes and arcs(i,j) ($\forall i, j \in M;\ \ i \neq j$), where each node represents a potential site to locate "K" warehouses ($K \leq M$). Let c(i,j) be defined as a specific "link cost function" associated with an arc(i,j), where $\forall i, j \in M;\ \ i \neq j$; $X_{i,j}$ and Y_m be two binary decision variables, where

$\forall X_{i,j}, Y_m \in \{0,1\}$.

(1) Please formulate this facility location problem in a generalized form of a deterministic linear programming model with the objective function of minimum total link costs subject to some constraints if necessary. Note that the rationales associated with each equation should be provided (10%).

(2) Let us further assume that the link cost function (*i.e.*, c(i,j)) is composed of cost-related and risk related items. What can be the elements that should be taken into account when determining cost-related and risk-related items in model formulation? (5%)

(3) Please explain any adjustments needed to re-formulate the proposed link cost-minimum model in the following scenarios: (a) transnational context, (b) link cost with uncertainty, and (c) dynamic link costs. (5%)

2. (10%)

Let a primal problem be formulated as follows:

$$\text{Max } z = \mathbf{C}^{\mathrm{T}}\mathbf{X}$$
$$\text{st.}\quad \mathbf{AX} \leq \mathbf{B}$$
$$\mathbf{X} \geq \mathbf{0}$$

where $\mathbf{C}$ is a $N \times 1$ coefficient vector ($\mathbf{C} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix}$); $\mathbf{X}$ is a $N \times 1$ decision-variable

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vector ($\mathbf{X}=\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix}$); **A** is a $M \times N$ coefficient matrix ($\mathbf{A}=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & \ddots & & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ a_{M1} & \cdots & \cdots & a_{MN} \end{bmatrix}$); and

B is a $M \times 1$ upper-bound vector ($\mathbf{B}=\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_M \end{bmatrix}$) defined for constraints. Let us further

define $\tilde{z}$ as the value of the objective function for the associated dual problem; and **Y**

be a $M \times 1$ decision-variable vector ($\mathbf{Y}=\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix}$) of the dual problem. Please (a)

reformulate the aforementioned primal problem into a dual problem in a vector form (5%), and (b) prove $\tilde{z} \geq z$ (5%).

3. (20%)

Let the components of Bayes decision be defined as follows:

(1) H_0 and H_1: Hypotheses, where $H_0 \cap H_1 = \phi$;

(2) $\mathbf{x}$: an observation vector containing random samples obtained from the observation space Ψ which is composed of two observation regions ψ_0 and ψ_1;

(3) D_0 and D_1: decisions assigning $\mathbf{x}$ to H_0 and H_1, respectively;

(4) Decision events interested:

(a) $D_0 \cap H_0 \equiv D_0H_0$ with probability $P(D_0H_0)=P(D_0/H_0)\cdot P(H_0)$;

(b) $M \equiv D_0 \cap H_1 \equiv D_0H_1$ (termed detection-missing event) with probability $P(D_0H_1)=P(D_0/H_1)\cdot P(H_1)$

(c) $F \equiv D_1 \cap H_0 \equiv D_1H_0$ (termed false alarm event) with probability $P(D_1H_0)=P(D_1/H_0)\cdot P(H_0)$

(d) $D \equiv D_1 \cap H_1 \equiv D_1H_1$ with probability $P(D_1H_1)=P(D_1/H_1)\cdot P(H_1)$

(5) c_{ij}: decision cost associated with the event D_iH_j, where $i, j \in \{0,1\}$.

Let $B(d)$ be defined as Bayes risk. Please answer the following questions:

(1) What does Bayes risk means? Furthermore, please also explain the relationship between Bayes risk and Bayes decision rule (5%).

(2) Please formulate $B(d)$ in a mathematical form using the decision elements defined above (5%)

(3) Based on $B(d)$ formulated above, please derive Bayes decision rule using the decision elements defined above (5%). Please feel free to define any additional variables/parameters if needed.

(4) Let $P(H_0)=1/2$, $P(H_1)=1/2$, $c_{10}=c_{01}=10$, $c_{11}=c_{00}=0$, $P_{H_0}(x)$ and $P_{H_1}(x)$ represent the probability density functions associated with H_0 and H_1, characterized by uniform distributions $U_0[0,10]$ and $U_1[2,14]$, respectively. Given the observation vector $\mathbf{x}=[5 \quad 3 \quad 9]$, please identify the hypothesis (H_0 or H_1) that might hold true using Bayes decision rule, where your explanations must also be provided (5%).

4. (25%)

Due to increased sales, a company is considering building 3 new distribution centers (DCs) to serve 4 regional sales areas. The annual cost to operate DC 1 is $500 (in thousands of dollars). The cost to operate DC 2 is $600 (in thousands of dollars.). The cost to operate DC 3 is $525 (thousands of dollars). Assume that the variable cost of operating at each location is the same, and therefore not a consideration in making the location decision.

The table below shows the cost ($ per item) for shipping from each DC to each region.

Region

DC	A	B	C	D
1	1	3	3	2
2	2	4	1	3
3	3	2	2	3

The demand for region A is 70,000 units; for region B, 100,000 units; for region C, 50,000 units; and for region D, 80,000 units. Assume that the minimum capacity for the distribution center will be 500,000 units.

Assume that Xij = quantity shipped from distribution i to region j, i = 1,2,3; j = 1, 2, 3, 4.

Assume that Yi = 0 or 1 where i = distribution center 1, 2 or 3.

(1) The constraint for distribution center 1 is ? (8%)

(2) The objective function is (8%)

(3) You have been asked to select at least 3 out of 7 possible sites for oil exploration. Designate each site as S_1, S_2, S_3, S_4, S_5, S_6, and S_7. The restrictions are :

Restriction 1. Evaluating sites S_1 *and* S_3 will prevent you from exploring site S_7.

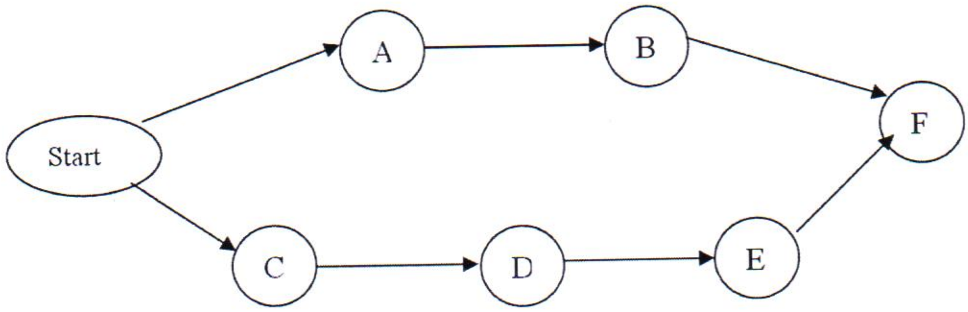
Restriction 2. Evaluating sites S_2 *or* S_4 will prevent you from assessing site S_5.

Restriction 3. Of all the sites, at least 3 should be assessed.

Assuming that S_i is a binary variable, write the constraint(s) for the second restriction (9%)

5. (25%)

The diagram below shows the activities on the nodes, and the table shows the normal time, crash time and cost for each activity, in days.



Activity	Normal Time	Crash Time	Cost per day to crash
A	6	6	---
B	10	8	$100
C	5	4	$300
D	4	1	$700
E	9	7	$500
F	2	1	$650

(1) Determine which activities should be crashed to shorten the project by 1 day. (8%)

(2) Determine which activities should be crashed to shorten the project by 2 days. (8%)

(3) Determine which activities should be crashed to shorten the project by 3 days. What is the cost? (9%)