國立台灣大學商學研究所博士班入學考試試卷（93 學年度）

科目　統計學

1. (10%) A new filtering device is installed in a chemical unit. Before its installation, a random sample yielded the following information about the percentage of impurity: $\overline{Y}_1 = 12.5$, $S_1^2 = 101.17$, and $n_1 = 8$. After installation, a random sample yielded $\overline{Y}_2 = 10.2$, $S_2^2 = 94.73$, $n_2 = 9$. (assume $\alpha = 0.05$)
   (a) (5%) Suppose that both samples have equal variances. To test if the means of two samples are the same, we use t test. What is the calculated t value? $|t \text{ value}| = ?$
   (b) (5%) What is the critical t value to reject the null hypothesis of equal means. t critical value = ?

2. (20%) A manufacturer of television sets is interested in the effect on tube conductivity of four different types of coating for color picture tubes. The following conductivity data are obtained:

Coating Type	Conductivity				$\overline{Y}_{i\bullet}$	S_i^2
1	143	141	150	146	145.00	15.3333
2	152	149	137	143	145.25	44.2500
3	134	136	132	127	132.25	14.9167
4	129	127	132	129	129.25	4.2500

To construct the ANOVA table, we need the following calculations:
   (a) (10%) Sum of square for Treatment (coating type) = ?
   (b) (5%) Sum of square for error = ?
   (c) (5%) Degrees of freedom for Treatment (coating type) = ?

3. (20%) Johnson and Leone describe an experiment to investigate warping of copper plates. The two factors studied were the temperature (X_1) and the copper content of the plates (X_2). The response variable was a measure of the amount of warping. The data were as follows:

	Copper Content (%)			
Temperature (°C)	40	60	80	100
50	17, 20	16, 21	24, 22	28, 27
75	12, 9	18, 13	17, 12	27, 31
100	16, 12	18, 21	25, 23	30, 23
125	21, 17	23, 21	23, 22	29, 31

(a) (10%) Complete the following two-way ANOVA table.

Source	SS	D.F.
X_1	156.09	
X_2	698.34	
X_1X_2		
Residual		
Total	1076.71	

(b) (10%) Complete the following ANOVA table that is obtained by fitting a regression model $\hat{Y} = b_0 + b_1X_1 + b_2X_2 + b_3X_1X_2$ to the data.

Source	SS	D.F.
Model	678.67	
Residual		
Total		

4. (20%) Roll a pair of four-sided dice, one red and one black. Let X equal the outcome on the red die and let Y equal the sum of the two dice.

(a) (5%) Define the joint p.m.f. on the space.

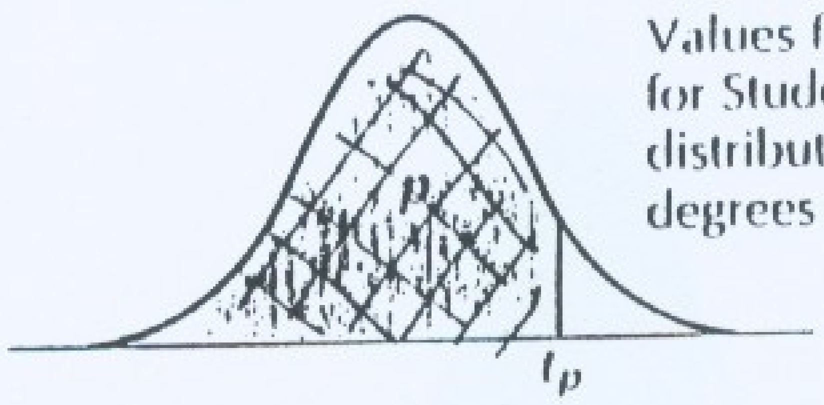
(b) (5%) Give the marginal p.m.f. of X in the margin.

(c) (5%) Give the marginal p.m.f. of Y in the margin.

(d) (5%) Are X and Y dependent or independent? Why?

5. (20%) Suppose that customers arrive at a checkout counter at an average rate of two customers per minute and that their arrivals follow the Poisson model.
   (a) (10%) Use the appropriate exponential distribution to find the probability that the next customer will arrive within 1 minute; within 2 minutes.
   (b) (5%) Use the exponential distribution to find the probability that the next customer will not arrive within the next 1.5 minutes.
   (c) (5%) Use the appropriate Poisson distribution to answer (b).

6. (10%) In April 1989, NBC aired a 1-hour special program titled"Black Athletes-Fact and Fiction," which attempted to explain why African-Americans dominate American professional and amateur sports. Some of the people interviewed believe that the reasons are genetic, while others point to social and cultural factors. In one test reported on the program, 1,200 black and white children up to age 6 were tested for motor skill development. The researchers found that in 30 separate tests, black children outperformed white children in 15, while white children were superior in 3. What conclusions can be drawn at the 5% significance level from the results of the 30 tests?

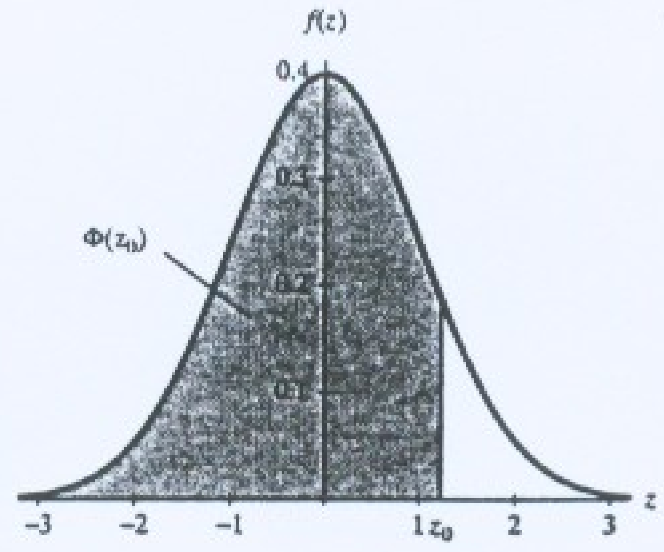


Values for t_p for Student's t distribution with ν degrees of freedom

TABLE A.3 Values of t for Various Probability Levels, p

ν	$t_{.60}$	$t_{.75}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.9975}$	$t_{.999}$	$t_{.9995}$
1	0.325	1.000	1.963	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.289	0.816	1.386	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.277	0.765	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.271	0.741	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.063	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.061	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.060	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.256	0.685	1.059	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.058	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.058	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.057	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.056	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.055	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.055	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.050	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
50	0.255	0.679	1.047	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.254	0.679	1.045	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
70	0.254	0.678	1.044	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
80	0.254	0.678	1.043	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
90	0.254	0.677	1.042	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
100	0.254	0.677	1.042	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
110	0.254	0.677	1.041	1.289	1.659	1.982	2.361	2.621	2.865	3.166	3.381
120	0.254	0.677	1.041	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

The Normal Distribution



$$P(Z \leq z) = \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

$$\Phi(-z) = 1 - \Phi(z)$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9545	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

α	0.400	.0300	0.200	0.100	0.050	0.025	0.020	0.010	0.005	0.001
z_{α}	0.253	0.524	0.842	1.282	1.645	1.960	2.054	2.326	2.576	3.090
$z_{\alpha/2}$	0.842	1.036	1.282	1.645	1.960	2.240	2.326	2.576	2.807	3.291

Poisson Probabilities

Tabulated values are $P(X \leq k) = \sum_{x=0}^{k} p(x)$. (Values are rounded to three decimal places.)

k	μ .10	.20	.30	.40	.50	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
0	.905	.819	.741	.670	.607	.368	.223	.135	.082	.050	.030	.018	.011	.007	.004	.002
1	.995	.982	.963	.938	.910	.736	.558	.406	.287	.199	.136	.092	.061	.040	.027	.017
2	1.000	.999	.996	.992	.986	.920	.809	.677	.544	.423	.321	.238	.174	.125	.088	.062
3	1.000	1.000	1.000	.999	.998	.981	.934	.857	.758	.647	.537	.433	.342	.265	.202	.151
4	1.000	1.000	1.000	1.000	1.000	.996	.981	.947	.891	.815	.725	.629	.532	.440	.358	.285
5						.999	.996	.983	.958	.916	.858	.785	.703	.616	.529	.446
6						1.000	.999	.995	.986	.966	.935	.889	.831	.762	.686	.606
7							1.000	.999	.996	.988	.973	.949	.913	.867	.809	.744
8								1.000	.999	.996	.990	.979	.960	.932	.894	.847
9									1.000	.999	.997	.992	.983	.968	.946	.916
10										1.000	.999	.997	.993	.986	.975	.957
11											1.000	.999	.998	.995	.989	.980
12												1.000	.999	.998	.996	.991
13													1.000	.999	.998	.996
14														1.000	.999	.999
15															1.000	.999
16																1.000
17																
18																
19																
20																

continued

k	μ 6.5	7.0	7.5	8.0	8.5	9.0	9.5	10	11	12	13
0	.002	.001	.001	.000	.000	.000	.000	.000	.000	.000	.000
1	.011	.007	.005	.003	.002	.001	.001	.000	.000	.000	.000
2	.043	.030	.020	.014	.009	.006	.004	.003	.001	.001	.000
3	.112	.082	.059	.042	.030	.021	.015	.010	.005	.002	.001
4	.224	.173	.132	.100	.074	.055	.040	.029	.015	.008	.004
5	.369	.301	.241	.191	.150	.116	.089	.067	.038	.020	.011
6	.527	.450	.378	.313	.256	.207	.165	.130	.079	.046	.026
7	.673	.599	.525	.453	.386	.324	.269	.220	.143	.090	.054
8	.792	.729	.662	.593	.523	.456	.392	.333	.232	.155	.100
9	.877	.830	.776	.717	.653	.587	.522	.458	.341	.242	.166
10	.933	.901	.862	.816	.763	.706	.645	.583	.460	.347	.252
11	.966	.947	.921	.888	.849	.803	.752	.697	.579	.462	.353
12	.984	.973	.957	.936	.909	.876	.836	.792	.689	.576	.463
13	.993	.987	.978	.966	.949	.926	.898	.864	.781	.682	.573
14	.997	.994	.990	.983	.973	.959	.940	.917	.854	.772	.675
15	.999	.998	.995	.992	.986	.978	.967	.951	.907	.844	.764
16	1.000	.999	.998	.996	.993	.989	.982	.973	.944	.899	.835
17		1.000	.999	.998	.997	.995	.991	.986	.968	.937	.890
18			1.000	.999	.999	.998	.996	.993	.982	.963	.930
19				1.000	.999	.999	.998	.997	.991	.979	.957
20					1.000	1.000	.999	.998	.995	.988	.975
21							1.000	.999	.998	.994	.986
22								1.000	.999	.997	.992
23									1.000	.999	.996
24										.999	.998
25										1.000	.999
26											1.000
27											
28											
29											