國立台灣大學商學研究所博士班入學考試試卷（98 學年度）

科目 作業研究　（考試時間 2 小時）

1. You are using the simplex method to solve the following linear programming problem.

Maximize $Z = 6x_1 + 5x_2 - x_3 + 4x_4$,

subject to

$3x_1 + 2x_2 - 3x_3 + x_4 \leq 120$

$3x_1 + 3x_2 + x_3 + 3x_4 \leq 180$

and

$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0.$

You have obtained the following final simplex tableau where x_5 and x_6 are the slack variables for the respective constraints.

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side
Z	(0)	1	0	$\frac{1}{4}$	0	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	Z^*
x_1	(1)	0	1	$\frac{11}{12}$	0	$\frac{5}{6}$	$\frac{1}{12}$	$\frac{1}{4}$	b_1^*
x_3	(2)	0	0	$\frac{1}{4}$	1	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	b_2^*

Please identify Z^*, b_1^*, and b_2^*. (24 points)

2. Ken and Larry, Inc., supplies its ice cream parlors with three flavors of ice cream: chocolate, vanilla, and banana. Due to extremely hot weather and a high demand for its products, the company has run short of its supply of ingredients: milk, sugar, and cream. Hence, they will not be able to fill all the orders received from their retail outlets, the ice cream parlors. Due to these circumstances, the company has decided to choose the amount of each flavor to produce that will maximize total profit, given the constraints on supply of the basic ingredients.

The chocolate, vanilla, and banana flavors generate, respectively, \$1.00, \$0.90, and \$0.95 of profit per gallon sold. The company has only 200 gallons of milk, 150 pounds of sugar, and 60 gallons of cream left in its inventory. The linear programming formulation for this problem is shown below in algebraic form.

Let C = gallons of chocolate ice cream produced,

V = gallons of vanilla ice cream produced,

B = gallons of banana ice cream produced.

Maximize Profit = 1.00 C + 0.90 V + 0.95 B,

subject to

Milk: $0.45\,C + 0.50\,V + 0.40\,B \leq 200$ gallons

Sugar: $0.50\,C + 0.40\,V + 0.40\,B \leq 150$ pounds

Cream: $0.10\,C + 0.15\,V + 0.20\,B \leq 60$ gallons

and

$C \geq 0, \quad V \geq 0, \quad B \geq 0.$

This problem was solved using the Excel Solver. The spreadsheet (already solved) and the sensitivity report are shown below. [Note: The numbers in the sensitivity report for the milk constraint are missing on purpose, since you will be asked to fill in these numbers in part (f).]

	A	B	C	D	E	F	G
1		Resource Usage Per Unit of Each Activity					
2			Activity				Resource
3	Resource	Chocolate	Vanilla	Banana	Totals		Available
4	Milk	0.45	0.5	0.4	180	≤	200
5	Sugar	0.5	0.4	0.4	150	≤	150
6	Cream	0.1	0.15	0.2	60	≤	60
7	Unit Profit	1	0.9	0.95	$341.25		
8	Solution	0	300	75			

Changing Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B8	Solution Chocolate	0	-0.0375	1	0.0375	1E+30
C8	Solution Vanilla	300	0	0.9	0.05	0.0125
D8	Solution Banana	75	0	0.95	0.021429	0.05

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E4	Milk Totals					
E5	Sugar Totals	150	1.875	150	10	30
E6	Cream Totals	60	1	60	15	3.75

For each of the following parts, answer the question as specifically and completely as is possible without solving the problem again on the Excel Solver. Note: Each part is independent (i.e., any change made to the model in one part does not apply to any other parts). (26 points) (***a***) What is the optimal solution and total profit? (4 points)

(***b***) Suppose the profit per gallon of banana changes to $1.00. Will the optimal solution change, and what can be said about the effect on total profit? (4 points)

(***c***) Suppose the profit per gallon of banana changes to 92¢. Will the optimal solution change, and what can be said about the effect on total profit? (4 points)

(***d***) Suppose the company discovers that 3 gallons of cream have gone sour and so must be thrown out. Will the optimal solution change, and what can be said about the effect on total profit? (4 points)

(***e***) Suppose the company has the opportunity to buy an additional 15 pounds of sugar at a total cost of $15. Should they? Explain. (5 points)

(***f***) Fill in all the sensitivity report information for the milk constraint, given just the optimal solution for the problem. Explain how you were able to deduce each number. (5 points)

3. There are n stocks we can invest in, and our current portfolio consists of $z_i \geq 0$ shares of stocks $i = 1, \dots, n$, whose current prices are $\$q_i$ per share of stock i. (The z_i's and q_i's are known, and the current value of our portfolio is $\sum_{i=1}^{n} q_i z_i$.)

We are considering re-balancing our portfolio by buying and selling stocks $i = 1, \dots, n$ at their current prices. We pay a transaction fee of $\$a$ for every share traded (bought or sold).

The price per share of stock i in the next period is a random variable P_i. The following data is available to us:

- Expected share prices, $E[P_i]$
- Covariances $Cov(P_i, P_j)$, $i, j = 1, \dots, n$.

Our goal is to maximize the expected value of the portfolio in the next period, subject to the following constraints:

1. The variance of the value of the portfolio in the next period should be at most σ^2
2. We cannot "short" a stock, i.e., we cannot sell more shares of a stock than we own
3. No additional cash investment can be made, i.e., all the stock purchases and transaction fees have to be paid for with the money received from selling other stocks in the portfolio. Moreover, one of the "stocks" is actually a riskless investment, so without loss of generality, we can assume that all money is reinvested.

Formulate the above as an optimization problem. Clearly define your variables and identify the constraints. (20 points)

4. You have *D* doctor-years (if one doctor works for one year, it is one doctor year) available over the *N* years to help the public health of an African country. The number of doctor-years committed to year i can be further broken down to three activities: doctoral-years vaccination v_i, education e_i, and working in clinic c_i. v_i, e_i, c_i must be nonnegative integers. Such a commitment will yield $L_i(v_i, e_i, c_i)$ lives saved in year i. Note that $L_i(v_i, e_i, c_i)$ is increasing in all arguments, and for (v_i, e_i, c_i), $i = 1, \dots, N$ to be a feasible arrangement, they must satisfy $\sum_{i=1}^{N}(v_i + e_i + c_i) = D$.

(a) Formulate a dynamic program to maximize total lives saved over the *N* years. (15 points)

(b) Solve the problem with *N=2, D=6*, and

$$L_1(v_1, e_1, c_1) = 2v_1 + v_1 e_1 + 3e_1^2 + 4\sqrt{c_1},$$
$$L_2(v_2, e_2, c_2) = v_2^2 + 6e_2 + 2c_2 + e_2 c_2. \qquad \text{(15 points)}$$