Introduction to Game Theory

- Description of a game
- Classification of games
- Extensive form and normal form
- Nash Equilibrium
- Subgame perfect Nash equilibrium

Description of a Game

- The set of players
- The strategies of players
- The payoffs of players as functions of players’ strategies

Equilibrium Concepts of Games

<table>
<thead>
<tr>
<th></th>
<th>Static</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete Information</td>
<td>Nash Equilibrium</td>
<td>Subgame perfect Nash Equilibrium</td>
</tr>
<tr>
<td>Incomplete Information</td>
<td>Bayesian Equilibrium</td>
<td>Perfect Bayesian Equilibrium</td>
</tr>
</tbody>
</table>

Two Ways to Formalize a Game

- Extensive form
- Normal form
**Extensive Form of a Game**
- Order of play
- Information and choices available to a player
- Payoffs of players
- Examples: game 1 and game 2

**The Normal Form of a Game**
- It shows the “Pure strategies” of players;
- It shows the payoffs as functions of players’ pure strategies
- Examples: game 1 and game 2

**Pure Strategies and Mixed Strategies**
- A pure strategy has to be a complete description of a player’s actions taken throughout the game;
- A mixed strategy is a probability distribution over the set of pure strategies.

**Game 1**
- Player 1 moves first and chooses L or R;
- Player 2, after observing player 2’s choice, chooses l or r.
- The payoff profiles are the same as in game 2 (to be shown later).
- What is the normal form of game 1?
Game 2

- Player 1 and player 2 move simultaneously;
- The strategies remain the same as in game 1.
- The payoff profiles are as follows:

<table>
<thead>
<tr>
<th>Player 1 \ Player 2</th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>(2, 0)</td>
<td>(2, -1)</td>
</tr>
<tr>
<td>R</td>
<td>(1, 0)</td>
<td>(3, 1)</td>
</tr>
</tbody>
</table>

Game 1

<table>
<thead>
<tr>
<th></th>
<th>1/2</th>
<th>(L, L)</th>
<th>(R, R)</th>
<th>(L, R)</th>
<th>(R, L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>2, 0</td>
<td>2, -1</td>
<td>2, 0</td>
<td>2, -1</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>1, 0</td>
<td>3, 1</td>
<td>3, 1</td>
<td>1, 0</td>
<td></td>
</tr>
</tbody>
</table>

The Equilibrium of Game 1

- Successive elimination of weakly dominated strategies
- Nash equilibrium (to be discussed)
- Perfect equilibrium (to be discussed)

Second-price Sealed Bid Auction

- N bidders with valuations $v_1 < v_2 < v_3 \ldots \ldots < v_n$;
- If bidder I wins, then
  $\pi_i = v_i - \max_{j \neq i} b_j$
- What would be the equilibrium?
Nash Equilibrium (NE)

A pure strategy Nash equilibrium is a set of actions such that no player, taking his opponents’ actions as given, wishes to change his own actions.

Theorem (Nash 1950)

Every finite strategic-form game has a mixed strategy equilibrium.

Theorem (Wilson)

Almost every finite game (finite number of players having finite number of strategies) has an odd number of NE’s in mixed strategies.

Prisoner’s Dilemma

<table>
<thead>
<tr>
<th>Player 1/Player 2</th>
<th>Cooperate</th>
<th>Not Cooperate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperate</td>
<td>(2,2)</td>
<td>(-3, 3)</td>
</tr>
<tr>
<td>Not Cooperate</td>
<td>(3, -3)</td>
<td>(-2, -2)</td>
</tr>
</tbody>
</table>
Example

<table>
<thead>
<tr>
<th>Player 1/2</th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>0,1</td>
<td>-1, 2</td>
</tr>
<tr>
<td>D</td>
<td>2,-1</td>
<td>-2, -2</td>
</tr>
</tbody>
</table>

* What would be the pure strategy Nash equilibria?
* What would be the mixed strategy Nash equilibrium?

Cournot Game with Simultaneous moves

- $P(Q) = 1 - Q$, where $Q = q_1 + q_2$
- Two firms choose $q_1$ and $q_2$ simultaneously.
- What would be the equilibrium?
- Reaction functions

Example: Matching Pennies

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>(1,-1)</td>
<td>(-1,1)</td>
</tr>
<tr>
<td>T</td>
<td>(-1,1)</td>
<td>(1,-1)</td>
</tr>
</tbody>
</table>

Targeting at Loyals or Switchers

<table>
<thead>
<tr>
<th>Segment</th>
<th>Proportion</th>
<th>Valuation for A</th>
<th>Valuation for B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loyal to A</td>
<td>0.5</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Loyal to B</td>
<td>0.2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Switchers</td>
<td>0.3</td>
<td>1.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Positioning Game

- Two firms choose the positions of their products along [0,1] simultaneously.
- Consumers’ ideal points are uniformly distributed.
- Each consumer chooses the product whose position is closest to his ideal point.
- Minimum or maximum differentiation?

Positioning Game (Cont.)

- What if there are 3 firms choosing their positions simultaneously?
- Is (1/2,1/2, 1/2) a Nash equilibrium?
- What if there are 4 firms?

Predation Game

- An entrant (E) decides to enter (In) or not;
- Then the incumbent (I) predates or not;
- I threatens to predate E whenever E is in.

What would be the equilibrium outcome?

- The payoffs for I and E are as follows:
  - E is out \(\rightarrow (3/4, 0)\)
  - E In and I predates \(\rightarrow (-1, -1)\)
  - E In and I not predate \(\rightarrow (0,1)\)
- What is the normal form of this game?
Perfect Equilibrium

- A refinement of Nash equilibrium for dynamic games
- (Selten 1965) A perfect equilibrium is a set of strategies such that in any subgame the strategies form a Nash equilibrium.
- The equilibrium can be derived by backward induction.

Subgames

- A subgame is the remainder of a game tree starting from some singleton information set.

Relation between IEDS and Perfect Equilibrium

- What is the perfect equilibrium for game 1?
- What if the payoff 3 is replaced by 2?

Cournot Game with sequential moves

- \( P(Q) = 1 - Q \), where \( Q = q_1 + q_2 \)
- Firm 1 moves first by choosing \( q_1 \)
- Firm 2 chooses \( q_2 \) after observing \( q_1 \)
- What would be the equilibrium?
- Comparison with the counterpart with simultaneous move
Games of Almost Perfect Information

- Repeated games
- Repeated game of prisoner’s dilemma.