Dynamic Games of Incomplete Information

Perfect Bayesian Equilibrium

A perfect Bayesian equilibrium is a set of strategies and posterior beliefs such that:

(P) Strategies are optimal given beliefs,
\[ a \in a^*(\mu^{\text{Bay}}(a)) \]

(B) Beliefs are obtained from strategies and observed actions using Bayes’ rule:
\[ \mu \in \mu^{\text{Bay}}(a^*(\mu)) \]

Three-door Example
Signaling Quality through a Low Price?

- Suppose a monopolist sells an experience good in two periods.
- Only previous buyers will consider buying in the second period.
- The quality of the product can be high (with prob. x) or low, which is known by the seller and will be known by buyers only after their consumption.

- The value of the high-quality product is $q$ and that of the low-quality one is 0.
- The cost of high-quality (low-quality) product is $c_1$ ($c_0$).
- Suppose that $x \cdot q < c_1$ (what does it mean?).
- Does there exist any separating equilibrium?
- Any pooling equilibrium?

Pooling Equilibrium

- The supporting posterior beliefs are
  \[
  \mu (H | p_1) = x \\
  \mu (L | p_1) = 1
  \]
- $p_1$ need satisfy the following conditions
  \[
  p_1 - c_0 \geq 0 \\
  p_1 - c_1 + \delta(\theta - c_1) \geq 0
  \]

Two Major Effects

- Nelson Effect
- Schmalensee Effect

What if the valuation to the low-quality product is $q > 0$?
Is it possible to signal high quality through a high price?
Example \((c_1 = 1/2; \ c_0 = 0; \ q_1 = 2, \ q_0 = 1; \ q_1 = 1; \ q_0 = 1/2; \ 1/3 < b < 1/2.)\)

- Two types of consumers, with taste parameters, \(q_1\) and \(q_0\), in proportion \(b\) and \(1-b\).
- Two types of a monopolist, with quality levels equal to \(q_1\) (high quality) and \(q_0\) (low quality), \(q_1 > q_0\).
- The unit costs for \(q_1\) and \(q_0\) equal \(c_1\) and \(c_0\).

Two-Period Reputation Game

- Two firms are in the market;
- Firm 1 can be sane type or crazy type, with probability \(x\) and \(1-x\).
- In period 1, firm 1 chooses to prey (P) or to accommodate (A), which yields profits \(P_2\) and \(D_2\) for firm 2, respectively, \(P_2 < 0 < D_2\).
- When sane, firm 1 obtains \(P_1\) and \(D_1\) by choosing P and A, respectively, \(P_1 < D_1\).

Two-Period Reputation Game (Cont.)

- When crazy, firm 1 always prefers to prey.
- In period 2, firm 2 chooses to stay or exit;
- If firm 2 exits, firm 1 obtains \(M_1\);
- If firm 2 stays, it gets \(P_2\) when firm 1 is crazy type and \(D_2\) when firm 1 is sane;
- The sane firm 1 obtains \(D_1\) when firm 2 stays.
- The discount factor is \(d\).

What would be the perfect Bayesian equilibria (PBE?)

- Separating equilibrium
- Semi-separating equilibrium
- Pooling equilibrium
Reconsider the Predation Game of Incomplete Information

The entrant (E) does not know the incumbent(I)’s type. E only knows that I is of sane type with prob. x. The payoffs for I and E are as follows:
- E is out $\rightarrow$ (3/4, 0)
- E In and I predates $\rightarrow$ (-1, -1) if $t_i$=sane
  (1/2, -1) if $t_i$=crazy
- E In and I not predate $\rightarrow$ (0,1)

What would be the equilibrium?

Chain Store Paradox

- An incumbent faces n potential entrants in its market;
- The entrants make entry decisions sequentially;
- At date i entrant i observes what happened in preceding i-1 markets and decides whether to enter.

Scenario 1

- If the predation game of complete information is repeated n times, what would be the perfect Nash equilibrium?

Scenario 2

- What would be the equilibrium for the predation game of incomplete information if n=2?
- What if n=3?
Sequential Bargaining

- A buyer and a seller negotiate over one unit of a product.
- The seller first offers $p_1$ which the buyer either accepts or rejects;
- If the buyer rejects $p_1$, then the seller offers $p_2$; both parties go away if $p_2$ is rejected.
- The value of the product to the buyer is either $b$ or $\overline{b}$, with equal probabilities.

- The value of the product for the seller is $s$.
- The discount factors are $d_b$ for the buyer and $d_s$ for the seller.
- $s < b < \overline{b}$.
- $b > \frac{\overline{b} + s}{2}$.
- What would be the perfect Bayesian equilibrium?

Spence’s Signaling Game

- Two types of workers: high productivity (H) and low productivity (L), which is the worker’s private information;
- The employer’s prior is $p(L)=a$, $p(H)=1- a$;
- A worker choose some education levels $e$ and demand wage $w$;
- The worker’s action $a_1 \in \{e, w\}$;

- The employer either accepts or rejects the worker’s offer.
- Any separating equilibrium?
- Any pooling equilibrium?