

國立台灣大學商學研究所博士班入學考試試卷 (100 學年度)

科目 統計學

第 1 頁 / 共 2 頁

- (10%) Suppose that  $X$  is a random variable for which  $E(X) = \mu$  and  $\text{Var}(X) = \sigma^2$ . Find  $E[X(X-1)] = ?$
- (10%) Three men  $A, B,$  and  $C$  shoot at a target. Suppose that  $A$  shoots three times and the probability that he will hit the target on any given shot is  $1/8$ ,  $B$  shoots five times and the probability that he will hit the target on any given shot is  $1/4$ , and  $C$  shoots twice and the probability that he will hit the target on any given shot is  $1/2$ . What is the expected number of times that the target will be hit?
- (10%) Suppose that customers arrive at a drive through window at an average rate of three customers per minute and that their arrival follow the Poisson model. Use the appropriate distribution to find the probability that the next customer will arrive within 1.5 minutes.
- (20%) A professor of statistics is trying to determine which of three statistical software is the best for his students. He believes that the time (in hours) it takes a student to master particular software may be influenced by gender. A  $3 \times 2$  factorial experiment with three replicates was designed, as shown below:

Software ( $X_1$ )	Gender ( $X_2$ )		$\bar{Y}_{i..}$
	Male	Female	
1	29, 24, 20	26, 32, 30	26.8333
2	32, 26, 21	23, 31, 25	26.3333
3	18, 20, 25	27, 22, 30	23.6667
$\bar{Y}_{.j.}$	23.8889	27.3333	$\bar{Y}_{...} = 25.6111$

To construct the ANOVA table, we need the following calculations:

- (5%) Mean square for factor  $X_1$  (software) = \_\_\_\_\_
- (5%) Mean square for factor  $X_2$  (gender) = \_\_\_\_\_
- (5%) Degrees of freedom for  $X_1 X_2$  = \_\_\_\_\_
- (5%) Degrees of freedom for error = \_\_\_\_\_

5. (10%) Let  $Y = X_1 + X_2 + \cdots + X_{16}$  be the sum of a random variable from the distribution with *p.d.f.*  $f(x) = x^2/4, -1 < x < 1$ . Find  $P(-0.4 \leq Y \leq 1.6)$ .
6. (10%) Let  $(X_1, X_2)$  be a random sample from Bernoulli distribution, which takes value of 1 with probability  $p$ . Suppose you will test  $H_0 : p = 2/3$  vs.  $H_1 : p \neq 2/3$ . You decide to reject the null hypothesis when  $|(X_1 + X_2)/3 - 2/3| \geq 1/9$ . Find the Type I error.
7. (10%) Let  $n = 200$  and the number of successes is 72 for a random sample drawn from a binomial population. Find
  - (a) The binomial proportion of successes.
  - (b) The variance of the binomial number of successes.
  - (c) The variance of the sample proportion of successes.
8. (10%) Consider a population (with 400 elements) follows a normal distribution with a mean of 50 and a standard deviation of 20. Suppose 25 elements are taken from this population. Find the probability that the mean of the sample is smaller than 48.
9. (10%) Consider 200 dice are tossed simultaneously. Let  $X_i$  be the number that shows on the  $i$ -th die. Find the probability that  $X_1 + X_2 + \cdots + X_{200} \geq 750$ .

試題請隨卷繳回