1. **(30 分)** 一名老闆雇用兩位員工（員工一與員工二）進行一投資計畫。

(1) 該計畫可能成功或失敗。若計畫成功，則其產生的收益為 10；若計畫失敗，則產生的收益為 0。

(2) 員工薪資的型式為：若計畫成功，則員工一得到 W_1，員工二得到 W_2；若計畫失敗，則員工得不到任何薪資。W_1 與 W_2 的值由老闆決定。

(3) 計畫成功的機率決定於員工是否努力。每一位員工都可自行決定努力或不努力。對員工來說，努力需花費的成本為 1，而不努力則不用付出任何成本。若兩位員工均不努力，則計畫成功的機率為 0；若兩位員工中僅有一位努力而另一位不努力，則計畫成功的機率為 0.3；若兩位員工均努力，則計畫成功的機率為 0.8。

(4) 進行順序為：(i) 老闆決定並宣布 W_1 與 W_2。(ii) 得知 W_1 與 W_2 的值之後，員工一決定是否努力。(iii) 得知 W_1 與 W_2 的值且觀察到員工一是否努力後，員工二決定是否努力。(iv) 計畫結果（成功或失敗）實現。若計畫成功，則老闆依約給員工薪資。

(5) 每位員工極大化自己的期望薪資(即計畫成功機率乘以成功時獲得的薪資)扣除努力成本，且只要期望薪資不低於努力成本則願意來工作。老闆極大化期望利潤（利潤為計畫收益扣除給員工的薪資）。

假設老闆已決定讓兩位員工都努力。在此假設下，求出對老闆最適的 W_1 與 W_2。請列出重要計算步驟並扼要解釋，否則不給分。

2. **(20 分)** 考慮一如下的 exchange economy。體系中有消費者一、二兩人及 X、Y 兩種財貨。兩位消費者的效用函數分別是：

消費者一：$U_1(X_1,Y_1) = \max\{2X_1, Y_1\}$.

消費者二：$U_2(X_2,Y_2) = X_2 Y_2$.

其中 X_1 與 X_2 分別是消費者一、二消費的財貨 X 數量，Y_1 與 Y_2 分別是消費者一、二消費的財貨 Y 數量，消費數量均為非負。max 為一函數，對任何兩實數 p 與 q，$\max\{p, q\}$ 為兩者之間較大的值。又整個經濟體系中，財貨 X 的總量為 6，財貨 Y 的總量為 8。

請找出所有 Pareto efficient 的 $((X_1,Y_1),(X_2,Y_2))$ 消費組合，將之畫在 Edgeworth Box 中。扼要解釋解題邏輯及步驟。

50分 **3.** A monopolistic firm (called R) wants to produce and sell two products X and Y to 3 consumers, A, B, and C. A consumer may consume either zero or one unit of each product. We shall refer to the maximum price that consumer i is willing to pay for one unit of product j as consumer i's *reservation value* for product j, and denote it by v_{ij}. The information about v_{ij} is summarized in the following table:

product/consumer	A	B	C
X	1	2	8
Y	9	7	2

R seeks to maximize its profit, and R has no fixed costs. Let c_x and c_y be the unit costs for producing respectively product X and product Y. Recall that consumer i gets 0 consumer surplus if he makes no purchase, and he gets consumer surplus $v_{ij} - P_j$ from buying 1 unit of product j if the price of product j is P_j. Assume that consumers want to maximize the total consumer surplus from buying the two products. R can do the following 3 things at the same time: (1) selling product X separately at price P_x; (2) selling product Y separately at price P_y; and (3) selling a bundle that consists of 1 unit of X and 1 unit of Y at price P_{xy}. (Setting a price equal to $+\infty$ is allowed.)

(i) Suppose that $c_x = c_y = 1$. What are the profit-maximizing P_x, P_y, and P_{xy} for R? What's R's maximum profit?

(ii) Suppose that $c_x = c_y = 2$. What are the profit-maximizing P_x, P_y, and P_{xy} for R? What's R's maximum profit?

4. A monopolistic firm (called M) can issue a coupon to consumers interested in buying its product Z. There are two types of consumers, type H and type L, each interested in buying 1 unit of Z. The populations of these two types of consumers are respectively 8,000 and 15,000. A type-i consumer has reservation value (for its definition see Problem 3 above) v_i for Z. Assume that $v_H = 4$, $v_L = 2$, and that M has no production costs. At date 0, M can choose a price P for Z, and issue a coupon with face value $d \geq 0$ to consumers. Consumers go shopping at date 1, and each consumer can choose to or not to carry the coupon from date 0 to date 1. If a consumer can carry and show the coupon to M at date 1, then the consumer can pay $P - d$ instead of P for 1 unit of Z. It costs $k_i > 0$ for a type-i consumer to carry a coupon from date 0 to date 1. Assume that $k_H = 1$ and $k_L = 0$.

(i) Assume that consumers cannot resell Z after buying it from M. What are the profit-maximizing P and d for M? What's M's maximum profit?

(ii) Assume that each type-L consumer can buy at most 1 unit of Z from M, and after buying Z, he can choose to consume it, or to resell it at the price $P - 0.5$ to anyone else interested in buying. What are the profit-maximizing P and d for M? What's M's maximum profit?