國立台灣大學商學研究所博士班入學考試試卷（99 學年度）

科目 作業研究 第 1 頁 / 共 3 頁

（考試時間 2 小時）

1. (*24 points*) A simple repairperson looks after both machines 1 and 2. Each time it is repaired, machine i stays up for an exponential time with rate λ_i, i = 1, 2. When machine i fails, it requires an exponentially distributed amount of work with rate μ_i to complete its repair. The repairperson will always service machine 1 when it is down. For instance, if machine 1 fails while machine 2 is being repaired, then the repairperson will immediately stop work on machine 2 and start on 1. What proportion of time is machine 2 down?

2. (*26 points*) A particle moves on a circle through points which have been marked 0, 1, 2, 3, 4 (in a clockwise order). At each step it has a probability p of moving to the right (clockwise) and $1 - p$ to the left (counterclockwise). Let X_n denote its location on the circle after the nth step. The process $\{ X_n, n \geq 0\}$ is a Markov chain.
   (a) Find the transition probability matrix.
   (b) Calculate the limiting probabilities.

3. (*24 points*) John owns a bicycle shop. Most of John's bicycle sales are customer orders; however, he also stocks bicycles for walk-in customers. He stocks three types of bicycles — road-racing, cross-country, and mountain. A road-racing bike costs $1,200, a cross-country bike costs $1,700, and a mountain bike costs $900. He sells road-racing bikes for $1,800, cross-country bikes for $2,100, and mountain bikes for $1,200. He has $12,000 available this month to purchase bikes. Each bike must be assembled; a road-racing bike requires 8 hours to assemble, a cross-country bike requires 12 hours, and a mountain bike requires 16 hours. He estimates that he and his employees have 120 hours available to assemble bikes. He has enough space in his store to order 20 bikes this month. Based on past sales, John wants to stock at least twice as many mountain bikes as the other two combined because mountain bikes sell better.

   Suppose a linear programming model for this problem is partially formulated and the result of sensitivity analysis is obtained as follows:

Maximize $Z = 600\,x_1 + c_2\,x_2 + c_3\,x_3$

Subject to

$1200\,x_1 + a_{12}\,x_2 + a_{13}\,x_3 \le b_1$ (Constraint A)

$x_1 + a_{22}\,x_2 + a_{23}\,x_3 \le 20$ (Constraint B)

$a_{32}\,x_1 + 12\,x_2 + a_{33}\,x_3 \le b_3$ (Constraint C)

$a_{42}\,x_1 + a_{42}\,x_2 - x_3 \le 0$ (Constraint D)

$x_1, x_2, x_3 \ge 0$

Adjustable Cells

Cell	Name	Fianl Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	X1	3	0	600	1E+30	290.9090909
C4	X2	0	-320	400	320	1E+30
D4	X3	6	0	300	900	600

Constraints

Cell	Name	Fianl Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E6	Constraint A	9000	0	12000	1E+30	3000
E7	Constraint B	9	0	20	1E+30	11
E8	Constraint C	120	30	120	40	120
E9	Constraint D	0	180	0	10	7.5

(a) Compete the formulation of the model, stating clearly about the meanings of three decision variables (x_1, x_2, x_3) and four constraints (A, B, C, D).

(b) Should John try to increase his budget for purchasing bikes, increase space to stock bikes, or increase labor hours to assemble bikes? Why?

(c) If John were to hire an additional worker for 30 hours at $10 per hour, how much additional profit would he make, if any?

(d) If John were to purchase a cheaper cross-country bike for $1,200 and sell it for $1,900, would this affect the original solution?

4. () A youth soccer club has contracted with Holiday Helpers, a local travel agency, to broker hotel rooms for out-of-town teams that have entered the club's Labor Day weekend soccer tournament. The agency has 12 teams it needs to arrange rooms for at 8 possible hotels. The following tables show the number of rooms each team needs, the number of rooms available at each hotel, the room rate at each hotel, and the maximum room rate each team wants to pay:

Team	Max Rate	Rooms Needed
1. Arsenal	$70	15
2. United	75	18
3. Wildcats	60	20
4. Rage	80	12
5. Rapids	110	17
6. Storm	90	10
7. Tigers	70	18
8. Stars	80	18
9. Comets	80	20
10. Hurricanes	65	16
11. Strikers	90	20
12. Bees	100	14

Hotel	Room Rate	Rooms Available
A. Holiday	$90	41
B. Roadside	75	26
C. Bates	55	38
D. Hampson	95	25
E. Tilton	100	26
F. Marks	80	38
G. Bayside	70	35
H. Harriott	80	52

(a) All of a team's rooms must be at the same hotel. Formulate a model and develop a solution for the agency to reserve rooms for as many teams as possible, according to their needs.

(b) The travel agency has requested that each team indicate three hotels it would prefer to stay at, in order of priority, based on price, location, and facilities. The teams' preferences are shown in the following table:

	Team											
Hotel Priority	1	2	3	4	5	6	7	8	9	10	11	12
1	C	B	C	F	D	F	C	F	H	C	A	E
2	G	G	--	H	E	A	G	H	F	--	H	A
3	--	C	--	B	A	H	--	B	B	--	F	D

Determine a revised hotel room allocation to assign rooms to all teams while reflecting their preferences to the greatest possible extent.